

$y = f(x) = 2^x$



A triangle with vertices labeled A, B, and C. Vertex A is on the left, vertex B is at the top right, and vertex C is at the bottom right. The triangle is formed by three line segments connecting these points.

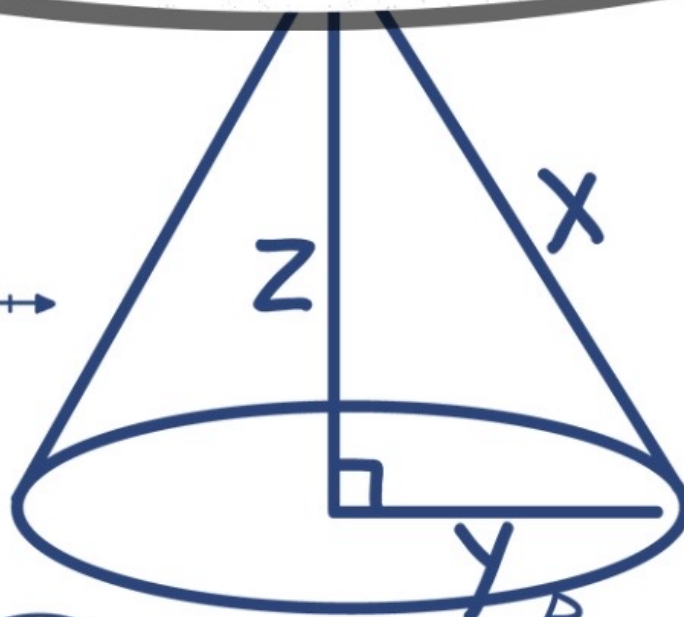
πr^2

$$\pi =$$



$$y = f(x) = 2^x$$

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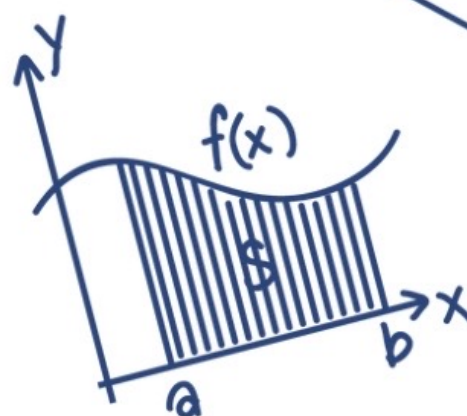


πr^2

$$\pi =$$



$$A = \frac{1}{2}bh$$



1.

$$P = \{ a, b, c, d \}$$

Write down all of the subsets of P that have 2 elements.

$\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$

2.

$\mathcal{E} = \{\text{even numbers between 1 and 29}\}$

$A = \{\text{prime numbers}\}$

$B = \{\text{factors of 24}\}$

$\mathcal{E} = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28\}$
 $A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$
 $B = \{1, 2, 3, 4, 6, 8, 12, 24\}$

List the elements of the set

(a) B'

$B' : 10, 14, 16, 18, 20, 22, 26, 28$

(b) $A \cap B$

2

3. A youth club has introduced three new activities

badminton (B), cookery (C) and drama (D).

Each of the 75 members of the youth club is asked to say in which of these activities they have participated.

Their answers showed that of the 75 members

all have participated in at least one of these activities

27 have participated in badminton and drama

31 have participated in badminton and cookery

23 have participated in cookery and drama

48 have participated in badminton

49 have participated in cookery

40 have participated in drama.

Let x be the number of members of the youth club who have participated in all three activities.

- (a) Using all this information, complete the Venn diagram opposite to show, in terms of x , the number of elements in each appropriate subset.

- (b) Find the value of x .

$$\begin{aligned}x + 56 &= 75 \\x &= 19\end{aligned}$$

- (c) Find

(i) $n(B \cap C') = 17$

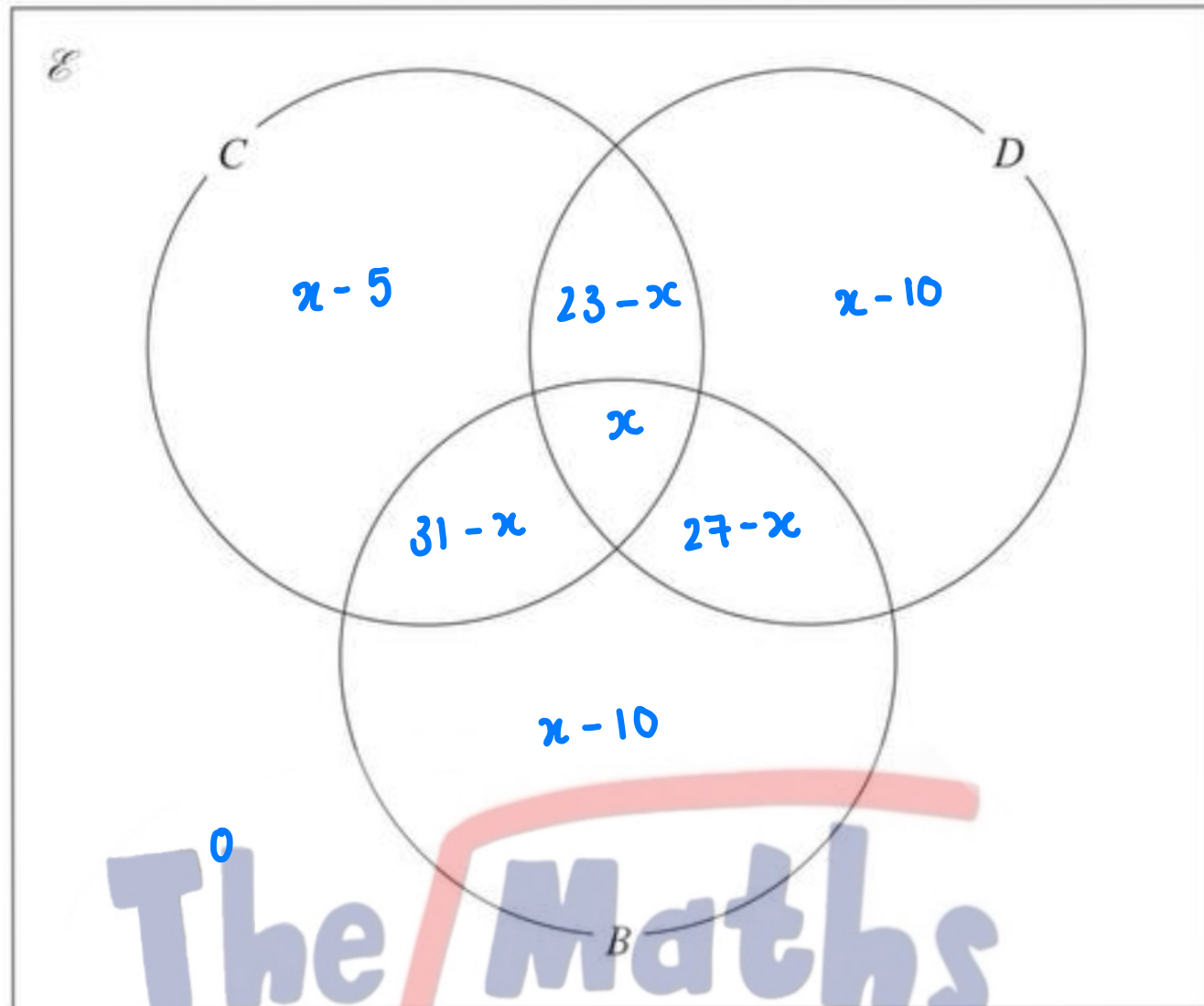
(ii) $n\left(\left[(B \cup C) \cap D\right]'\right) = 44$

One of the members of the youth club is picked at random.

Given that this member has participated in cooking,

- (d) find the probability that this member has not participated in any other activity.

$$\frac{19 - 5}{49} = \frac{14}{49}$$



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4. 90 students from a sixth form college were each asked the following question.

“Do you study any of Biology (B), Chemistry (C) or Physics (P)?”

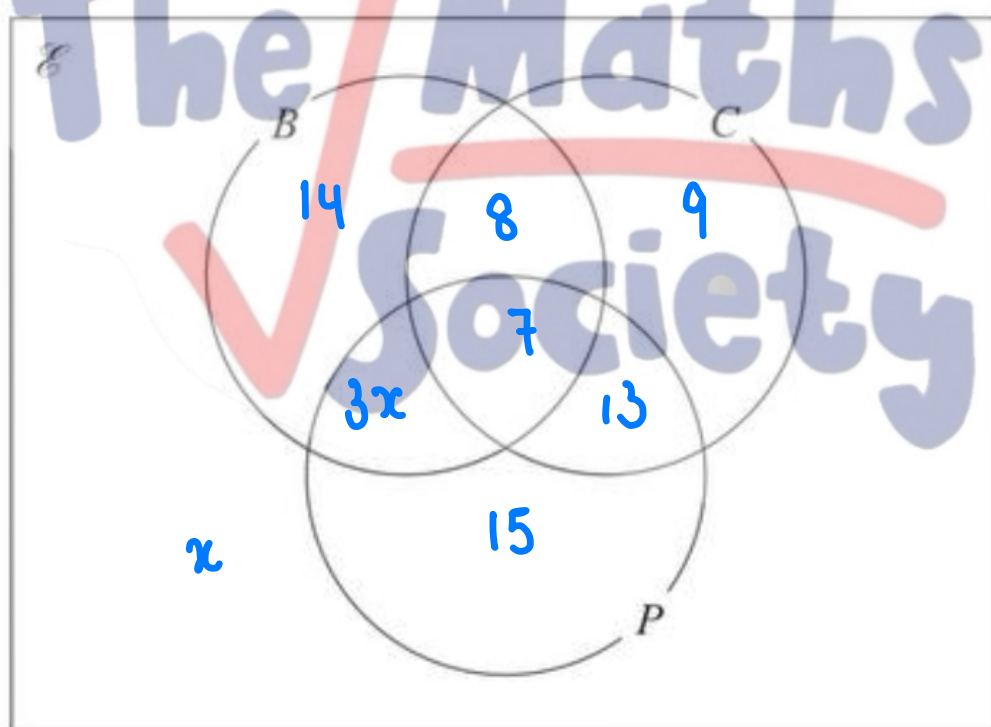
Of these 90 students

- 7 study all three subjects
- 15 study Biology and Chemistry
- 20 study Chemistry and Physics
- 37 study Chemistry
- 14 study Biology only
- 15 study Physics only

The number of these students who study Biology and Physics but not Chemistry is three times the number of these students who study none of these three subjects.

Let x be the number of these students who study none of these three subjects.

- (a) Show all this information on the Venn diagram, giving the number of students in each appropriate subset, in terms of x where necessary.



- (b) Find the value of x .

$$14 + 8 + 9 + 7 + 13 + 15 + 3x + x = 90$$

$$x = 6$$

- (c) Find

- (i) $n(B' \cap C) = 22$
- (ii) $n(B \cup C \cup P) = 84$
- (iii) $n(B \cap C \cap P') = 8$

One of the students is to be chosen at random.
Given that this student studies Chemistry,

- (d) find the probability that this student also studies Physics.

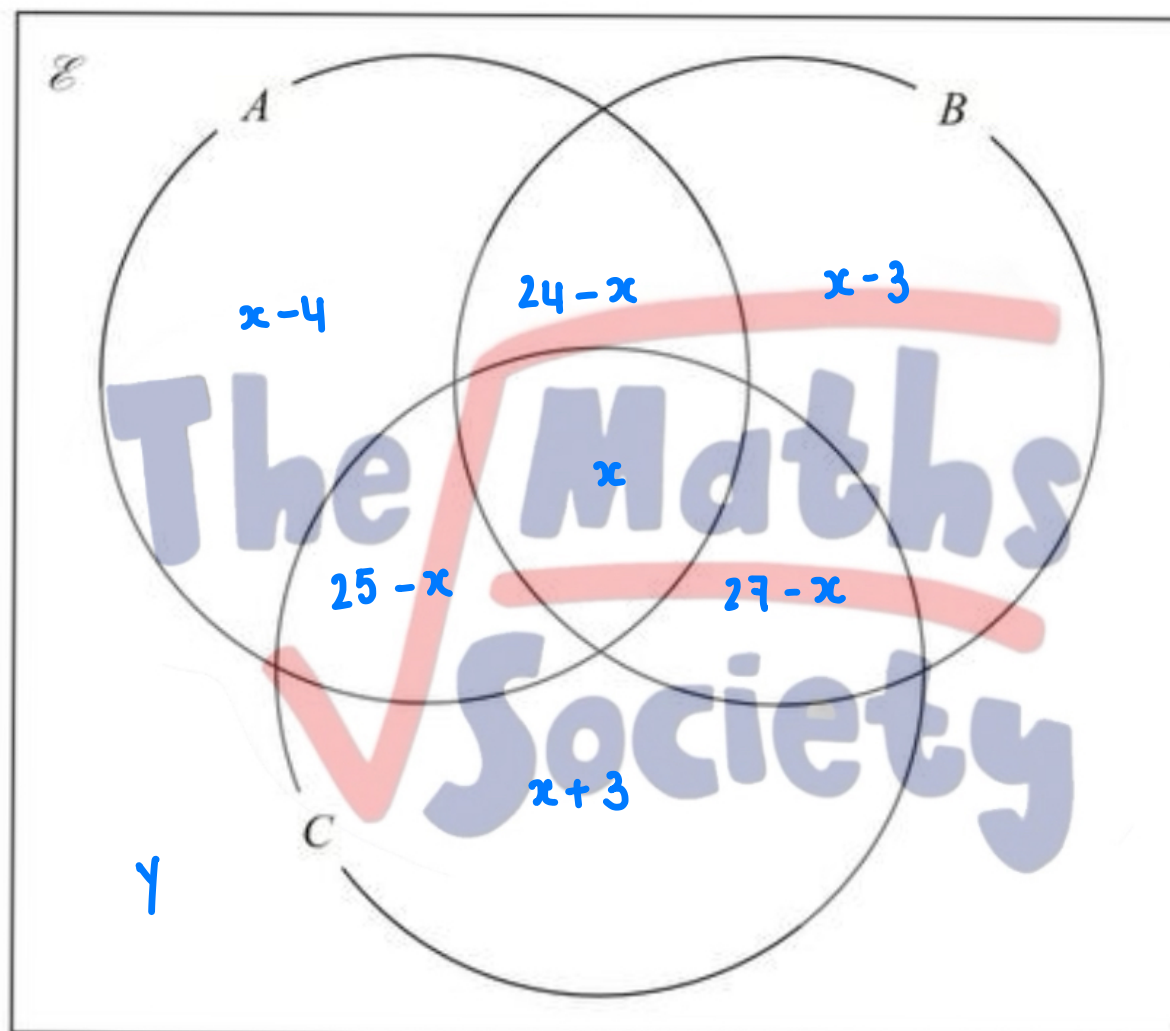
$$\frac{20}{37}$$

5. $\mathcal{E}$ is the universal set and A , B and C are three sets.

Here is information about these sets.

$$\begin{array}{lll} n(A) = 45 & n(B) = 48 & n(C) = 55 \\ n(A \cap B) = 24 & n(A \cap C) = 25 & n(B \cap C) = 27 \\ n(A \cap B \cap C) = x & n(A' \cap B' \cap C') = y & \end{array}$$

- (a) Show all this information on the Venn diagram, giving the number of elements in each appropriate subset, in terms of x or y .



$$x - 4 + 24 - x + x - 3 + 25 - x + x + 27 - x + x + 3 + y = 100$$

$$y = 28 - x$$

Given that $n(\mathcal{E}) = 100$

- (b) find an expression for y in terms of x .
Give your answer in its simplest form.

- (c) Using your Venn diagram and your answer to part (b), find the greatest possible value of x .
Give reasons for your answer.

$$24 - x \geq 0 \quad 24$$

x cannot be negative

An element is selected at random from the 100 elements in the universal set.

- (d) Find the probability that this element is in the set $(B \cap C') \cup (B' \cap C)$.

$$\frac{48 - 27}{100} + \frac{55 - 27}{100} = \frac{49}{100}$$

6. $\mathcal{E}$ is the universal set and A , B and C are three sets such that

$$\mathcal{E} = \{\text{even numbers between 5 and 31}\}$$

$$A = \{\text{factors of 24}\}$$

$$B = \{8, 16\}$$

$$C = \{\text{multiples of 6}\}$$

The Venn diagram on the opposite page can be used to show these sets.

(a) Complete the Venn diagram for the sets $\mathcal{E}$, A , B and C

List the elements of the set

(b) $A \cap C = 6, 12, 24$

(c) $(A \cup B \cup C)' = 10, 14, 20, 22, 26, 28$

Find

(d) $n([A \cup B]') = 8$

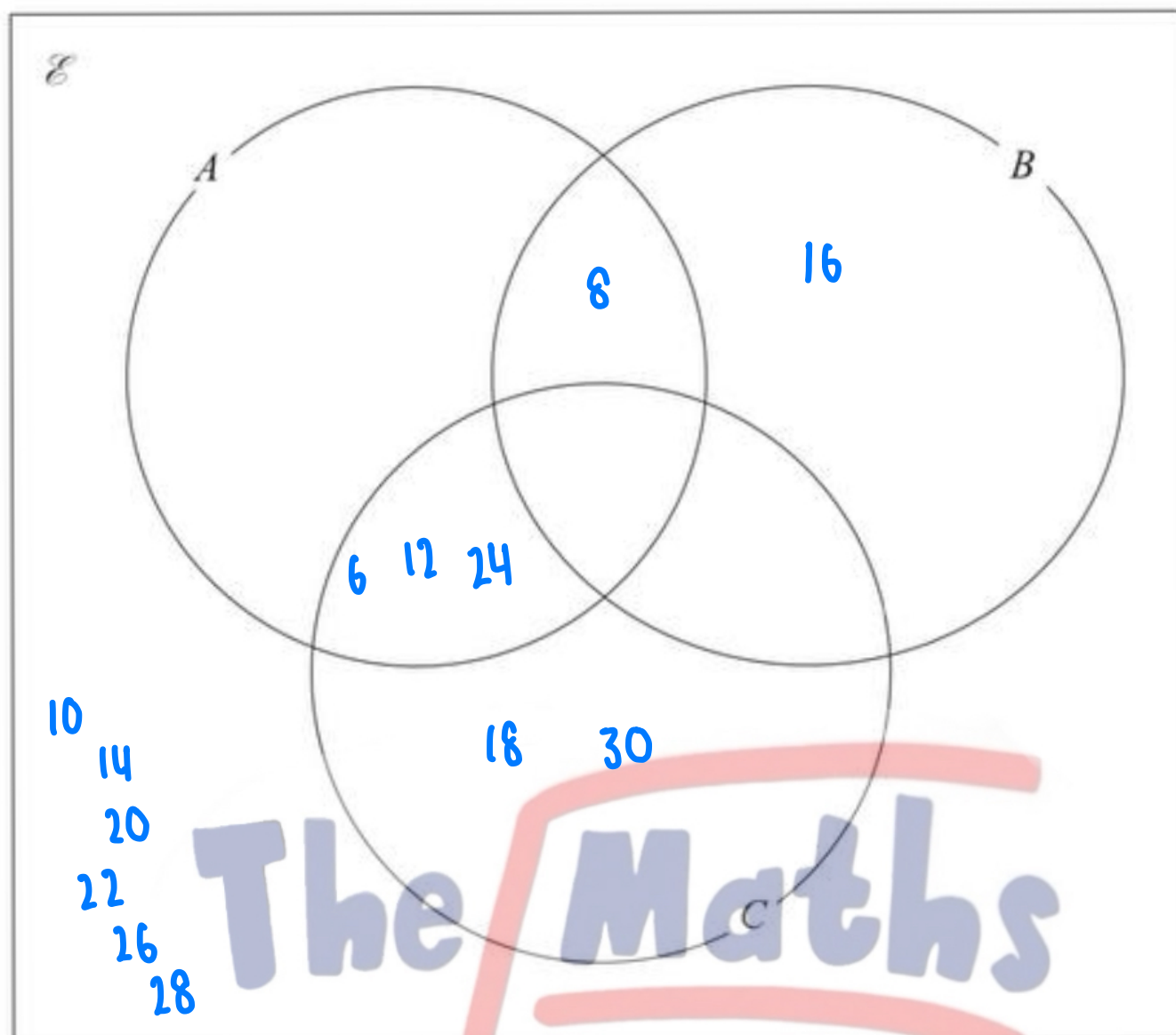
(e) $n([A \cap B] \cup C) = 6$

A number is selected at random from $\mathcal{E}$

(f) Find the probability that the number is in set B

total no. = 13

$\therefore \frac{2}{13}$



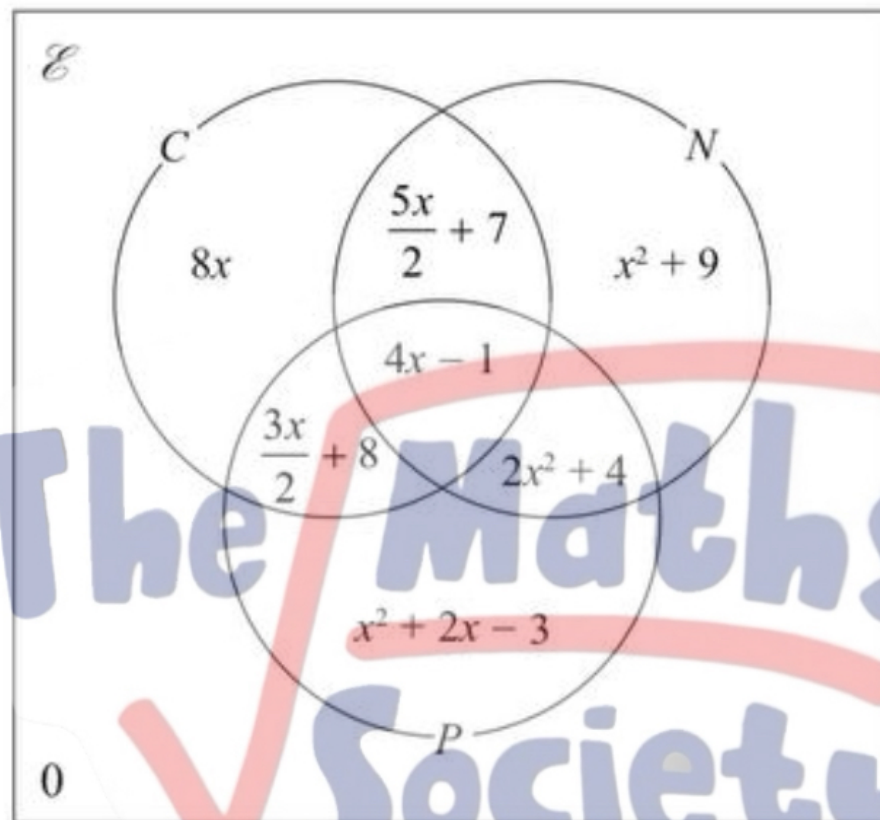
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7. The principal of a college wishes to introduce three new evening classes.

The classes are Cake decorating (C), Needlework (N) and Photography (P).

In a survey, 160 people who attend evening classes at the college were asked which of these new evening classes they would like to attend.

Information about the results of the survey is shown in the Venn diagram. The expression in terms of x , where x is a positive integer, in each subset represents the number of people in that subset.



- (a) (i) Show that $2x^2 + 9x - 68 = 0$

$$4x^2 + 18x + 24 = 160$$

$$2x^2 + 9x - 68 = 0$$

- (ii) Hence find the value of x .

$$(2x+17)(x-4) = 0 \quad \therefore x = 4$$

One of the people surveyed is selected at random.

Given that this person would like to attend Photography,

- (b) calculate the exact value of the probability that this person would also like to attend Cake decorating but not Needlework.

$$\frac{3}{2}x + 8$$

$$\left(\frac{3}{2}x + 8\right) + (4x - 1) + (2x^2 + 4) + (x^2 + 2x - 3)$$

$$x = 4$$

$$\frac{\frac{3 \times 4}{2} + 8}{3 \times 16 + 7.5 \times 4 + 8} = \frac{7}{43}$$

8. Some college students were each asked which of the subjects Mathematics (M), Physics (P) and Chemistry (C) they were studying.

Here is some information about their answers.

$$n(M \cap P) = 21$$

$$n(M \cap C) = 24$$

$$n(P \cap C) = 25$$

$$n(M) = 43$$

$$n(P) = 50$$

$$n([M \cup P \cup C]') = 25$$

$$n([M \cup P]' \cap C) = 8$$

$$n(M \cap P \cap C) = x, \text{ where } x \text{ is a positive integer.}$$

- (a) Use all the given information to complete the Venn diagram on the opposite page, giving the number of elements in each appropriate subset, in terms of x where necessary. (4)

Given that $n(C) = 40$

- (b) find the total number of college students that were asked. (4)

One of these college students is to be chosen at random.
Given that the college student studies Chemistry,

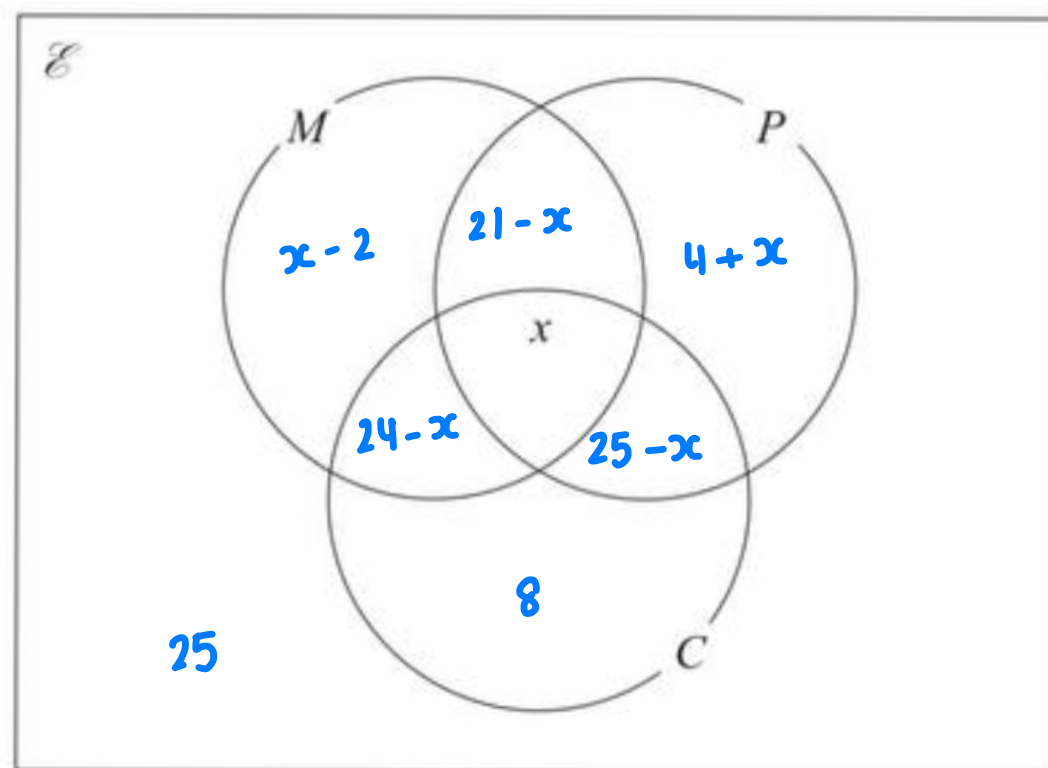
- (c) find the probability that this student also studies Physics. (2)

$$b) 24 - x + x + 25 - x + 8 = 40$$

$$x = 17$$

$$(17 - 2) + (21 - 7) + (4 + 17) + (24 - 17) + (17) + (25 - 17) + 8 + 25 = 105$$

$$c) \frac{25}{40} = \frac{5}{8}$$



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9. $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
 $A = \{2, 3, 5, 6, 9, 10\}$
 $B = \{3, 4, 6, 9, 12\}$

List the members of

(a) $A \cap B$

$= 3, 6, 9$

(b) $A \cup B$

$= 2, 3, 4, 5, 6, 9, 10, 12$

(c) A'

$1, 4, 7, 8, 11, 12$

$C = \{5, 6, 7\}$

(d) Write down all the subsets of C.

$\emptyset \{5\} \{6\} \{7\} \{5, 6\} \{5, 7\} \{6, 7\} \{5, 6, 7\}$

10. All 66 students in a drama group take part in at least one of the four activities given below.

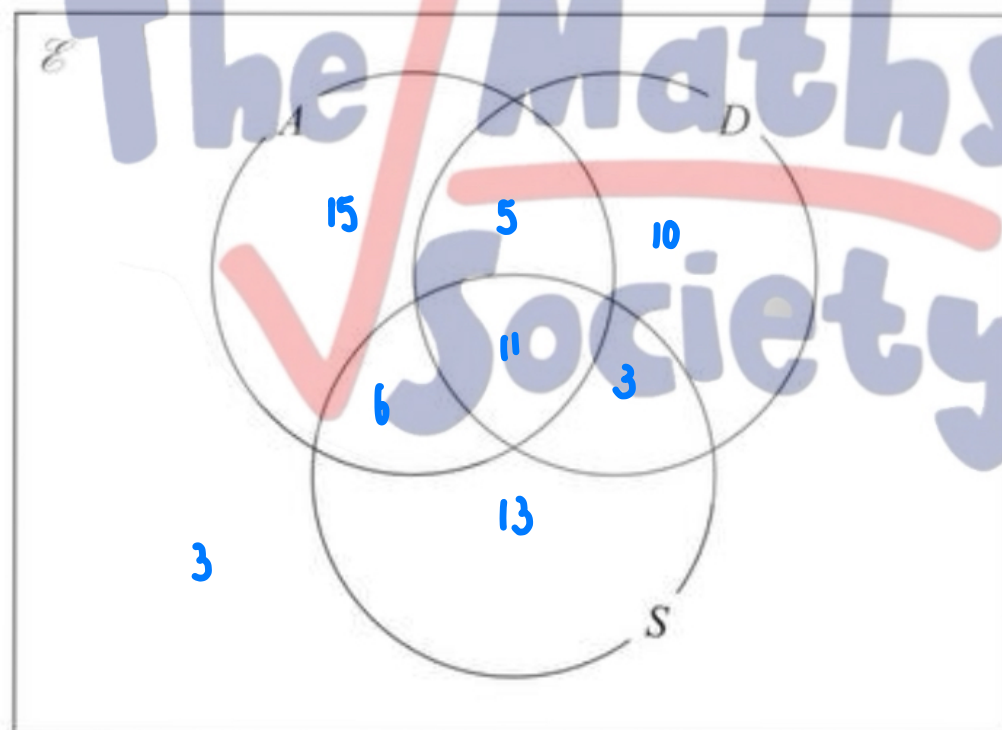
acting (A) dancing (D) singing (S) stage management (M)

Of these 66 students

- 11 take part in acting, dancing and singing
- 17 take part in acting and singing
- 14 take part in dancing and singing
- 16 take part in acting and dancing
- 33 take part in singing
- 37 take part in acting
- 29 take part in dancing

Any student who takes part in stage management does **not** take part in acting, dancing or singing.

- (a) Show all this information on the Venn diagram, giving the number of students in each subset.



- (b) Find
- (i) $n(M) : 3$
 - (ii) $n([A \cap D'] \cup S) : 15 + 6 + 11 + 3 + 13 = 48$
 - (iii) $n([A \cap D \cap S']') : 66 - 5 = 61$

One of the students in the drama group is selected at random.

Given that this student takes part in dancing,

- (c) find the probability that this student also takes part in

- (i) stage management,

0

- (ii) singing.

$$\frac{11 + 3}{10 + 11 + 3 + 5} = \frac{14}{29}$$

11. $\mathcal{E}$ is the universal set and A , B and C are three sets.

$$\mathcal{E} = \{p, q, r, s, t\} \quad A = \{q, r, s\} \quad B = \{p, q, t\}$$

(a) List the members of

(i) $A \cap B$ $= \{q\}$

(ii) $A \cup B$ $= \{p, q, r, s, t\}$

(iii) $A' \cap B$ $= \{p, t\}$

Given that $A \cap C = \{r\}$

(b) write down all the possibilities for set C

$$\{r\}, \{p, r\}, \{r, t\}, \{p, r, t\}$$

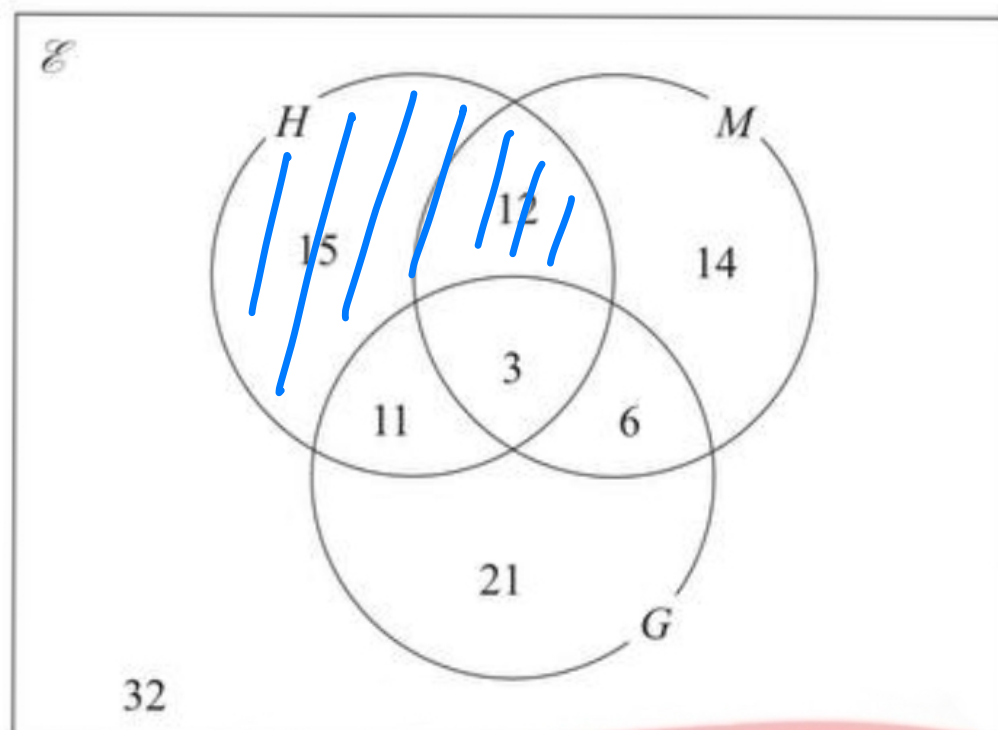
One of the possibilities for set C is selected at random.

(c) Find the probability that this set C is such that $B \cap C = \emptyset$

$$\{r\} \therefore \frac{1}{4}$$

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12. The Venn diagram shows information about the numbers of students in a school who study History (H), Music (M) and Geography (G).



- (a) Find the number of students who study Geography.
41
- (b) Find the number of students who study exactly two of the subjects History, Music and Geography.
29
- (c) On the Venn diagram above, shade the region that represents the set $H \cap G'$

$\mathcal{E}$ is the universal set and A and B are two sets such that

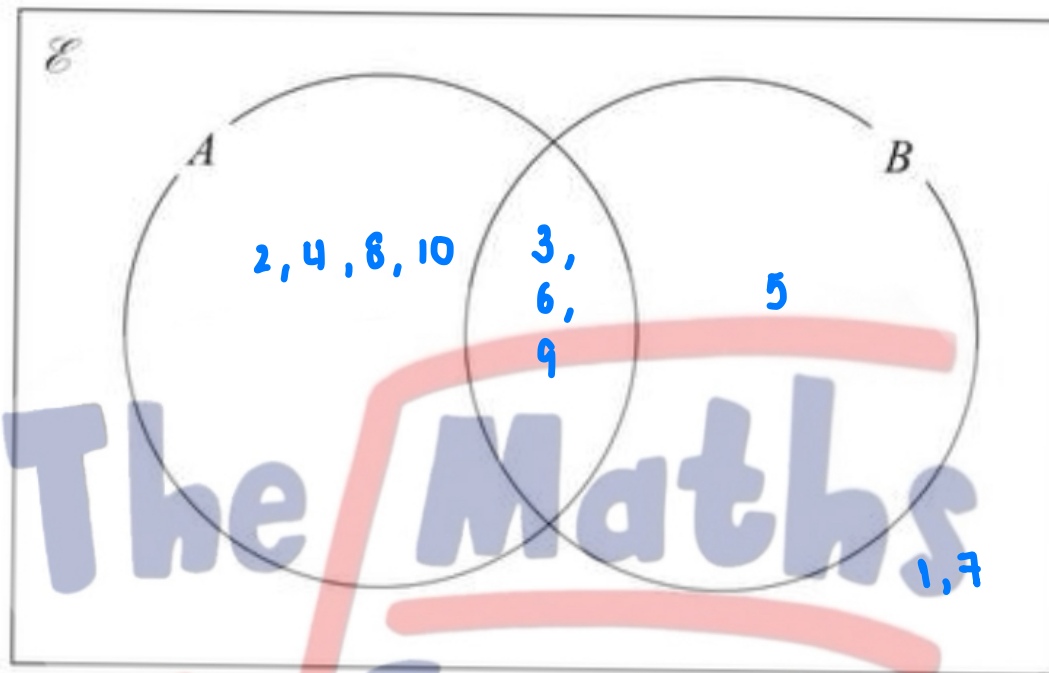
$$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A \cap B = \{\text{multiples of } 3\}$$

$$A \cap B' = \{2, 4, 8, 10\}$$

$$(A \cup B)' = \{1, 7\}$$

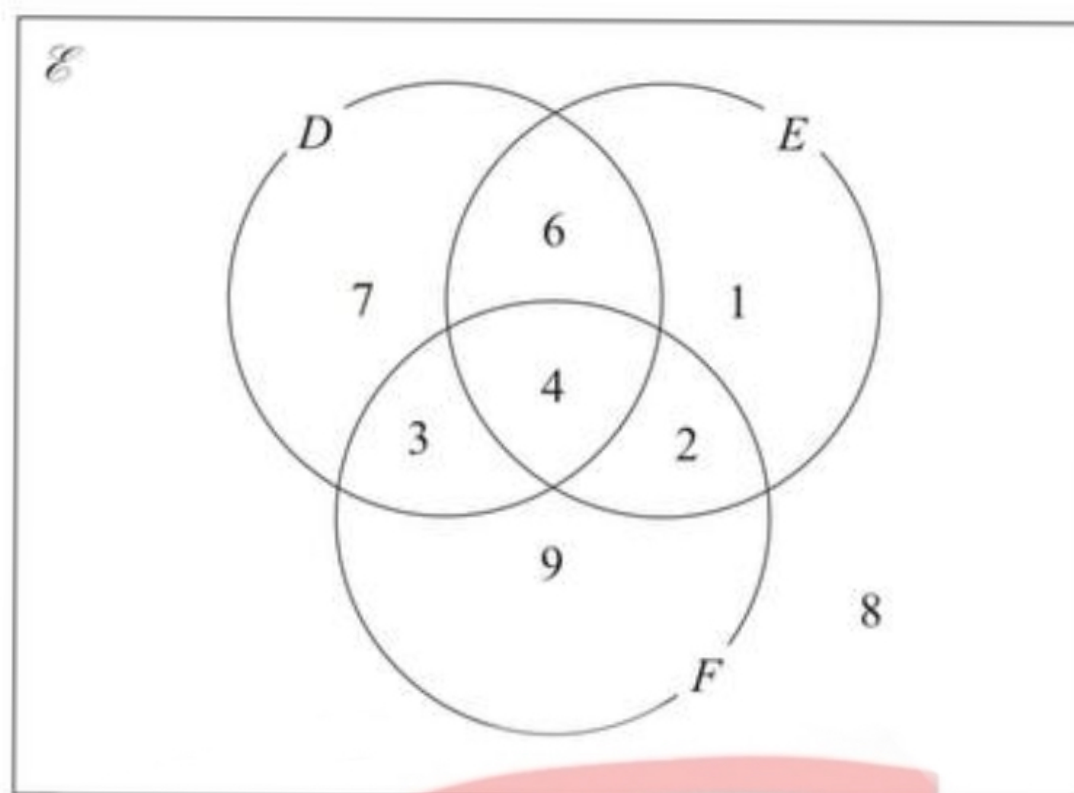
(d) Use the information about the sets $\mathcal{E}$, A and B to complete the Venn diagram below.



(e) Find $n(B)$

4

13. The Venn diagram shows the sets D , E and F where the numbers represent **numbers** of elements.



(a) Find $n(D \cup E)$

$$= 7 + 3 + 6 + 4 + 1 + 2 = 23$$

(b) Find $n(D \cap E')$

$$= 10$$

(c) Find $n(F')$

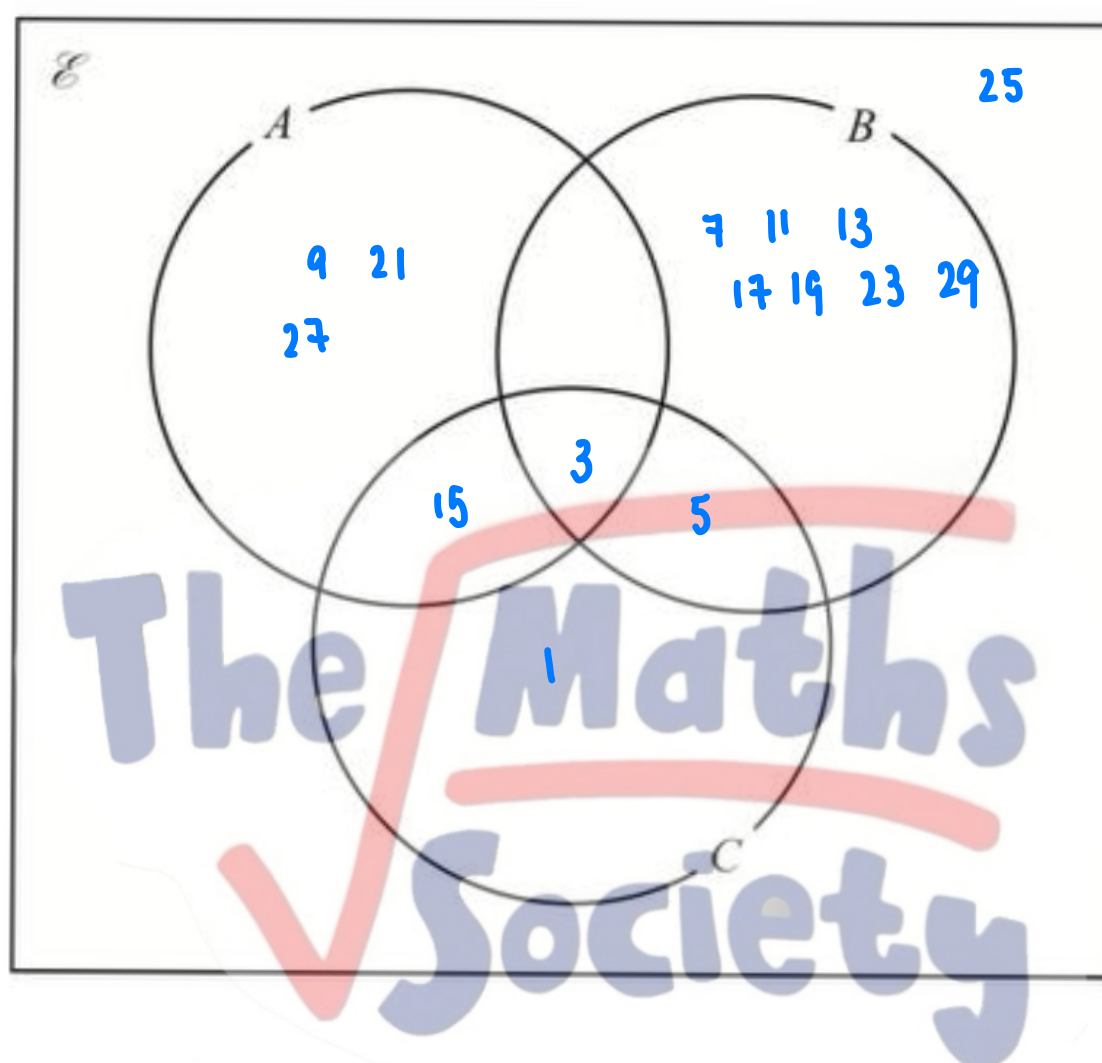
$$= 22$$

(d) Find $n([D \cup F] \cap E')$

$$19$$

14. $\mathcal{E} = \{\text{odd numbers between 0 and 30}\}$
 $A = \{\text{multiples of 3}\}$
 $B = \{\text{prime numbers}\}$
 $C = \{\text{factors of 30}\}$

(a) Complete the Venn diagram for this information showing the position of each of the numbers in the universal set.



- (b) Find (i) $n([A \cup C] \cap B) = 2$
(ii) $n([B \cap C'] \cup A') = 10$

A number is chosen at random from the universal set, $\mathcal{E}$

(c) Write down the probability that the number is in the set $C \cap A'$

$$\frac{2}{15}$$

Given that the number chosen from $\mathcal{E}$ is a multiple of 3

(d) find the probability that the number is also a factor of 30

$$\frac{2}{5}$$